Published by Institute of Physics Publishing for SISSA/ISAS

Received: *July 8, 2002*
Revised: *August 23, 2002*
Accepted: *September 24, 2002*

Construction of the vacuum string field theory on a non-BPS Brane

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Abstract: In the framework of the Sen conjectures a construction of vacuum superstring field theory on a non-BPS brane is discussed. A distinguished feature of this theory is a presence of a ghost kinetic operator mixing GSO± sectors. A candidate for such kinetic operator with zero cohomology is discussed.

Keywords: Tachyon Condensation, String Field Theory.

*On leave from Steklov Mathematical Institute.

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http://jhep.sissa.it/archive/papers/jhep092002050/jhep092002050.pdf

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1. Introduction

During the last year the bosonic vacuum string field theory (VSFT) proposed to describe physics around the bosonic tachyon vacuum [1] has been investigated in many papers [2]–[34]. VSFT action has the same form as the original Witten SFT action [35], but with a new differential operator $\mathcal{Q}$ (for a review of SFT see [36, 37, 38]). The absence of physical open string excitations around the tachyon vacuum [39]–[43] supports a suggestion [1] that after some field redefinition $\mathcal{Q}$ can be written as a pure ghost operator. Under this assumption solutions to VSFT equation of motion admit a factorized form with the projector-like matter part. Solutions of projector equation have been discussed in many details in the references [2]–[8]. This equation is similar to the non-commutative soliton equations in the large non-commutativity limit [44].

A generalization of VSFT to superstrings has been discussed in [1] and more recently in [45] and [46] in the context of cubic SSFT [49, 50] and non-polynomial SSFT [47], respectively. Open fermionic string in the NSR formalizm has a tachyon in the GSO− sector that leads to a classical instability of the perturbative vacuum in the theory without supersymmetry. It has been proposed [39] to interpret the tachyon condensation in the GSO− sector of the NS string as a decay of unstable non-BPS D9-brane.

The cubic action unifying NS GSO± sectors was constructed [48] as a generalization of the cubic action for GSO+ sector [49, 50]. As in the bosonic case the vacuum superstring field theory (VSSFT) is obtained by a shift of string field $\hat{A} = (A_+, A_-)$, which describes both GSO+ and GSO− sectors, by the tachyon vacuum $\hat{A}_0 = (A_{0,+}, A_{0,-})$. This shift leads to the new (shifted) BRST charge, that inevitably has matrix form. Assuming the statements of Sen conjectures we can also think that after a proper field redefinition the shifted BRST charge acts non-trivially only in (super)ghost sector. In this case VSSFT equations of motion factorizes into a matter part and a ghost part. But the factorization in the ghost part is a bit different as compare to one in the bosonic VSFT. In the matter part we will have standard equations for the projectors. Fermionic projectors, such as the sliver, have been constructed recently in [45, 46].

In the present paper we discuss a possible form of the vacuum ghost kinetic operator $\hat{\mathcal{Q}}$ in cubic VSSFT. Within the level truncation scheme it has been shown [48] that the tachyon potential in the cubic open SFT has a non-trivial minimum with $A_{0,+} \neq 0$ and $A_{0,-} \neq 0$.[1] Therefore, there is a reason to assume that the vacuum ghost kinetic operator $\hat{\mathcal{Q}}$:

$$\hat{\mathcal{Q}} = \begin{pmatrix} \mathcal{Q}_{\text{odd}} & \mathcal{Q}_{\text{even}} \\ -\mathcal{Q}_{\text{even}} & -\mathcal{Q}_{\text{odd}} \end{pmatrix}, \tag{1.1}$$

mixes GSO± sectors, i.e. with $\mathcal{Q}_{\text{even}} \neq 0$. If $\mathcal{Q}_{\text{even}}$ would be zero, we can take $\mathcal{Q}_{\text{odd}}$ to be the ghost kinetic operator used in the bosonic VSFT [24]. But since it is not the case we have to search for another expression for $\mathcal{Q}_{\text{odd}}$.

To be derivatives of the star algebra operators $\mathcal{Q}_{\text{odd}}$ and $\mathcal{Q}_{\text{even}}$ should satisfy some identities, in particular, $\langle \mathcal{I}|\mathcal{Q}_{\text{odd}} = 0$ and $\langle \mathcal{I}|\mathcal{Q}_{\text{even}} = 0$, here $\mathcal{I}$ is the identity of the star algebra [52]. Both $\langle \mathcal{I}|\mathcal{Q}_{\text{odd}}$ and $\langle \mathcal{I}|\mathcal{Q}_{\text{even}}$ are singular [52], therefore these expressions require some regularization and we will discuss it in section 3.2.

Note that to come back from VSSFT to the perturbative vacuum one has to study solutions of the VSSFT equations with non-zero components in both GSO+ and GSO− sectors (see figure 1).

The paper is organized as follows. In section 2 we describe cubic vacuum superstring field theory on the non-BPS D-brane. Construction of the kinetic operators is presented in section 3. Appendix A contains some technical details about matrix representation of the cubic action. Appendix B contains the construction of non-polynomial Vacuum SSFT on a non-BPS brane.

[1] Note, that the pure GSO+ slightly modified theory has a non-trivial saddle-point $A_{0,+} \neq 0$ at list at the first truncated levels [53]. However in this case there is no reason to assume that the corresponding $\mathcal{Q}$ is pure ghost.

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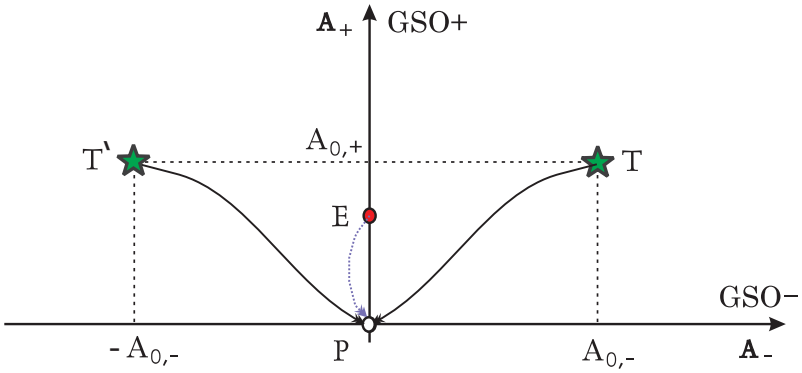

Figure 1: Scheme of vacua in SSFT. The point P denotes the perturbative vacuum, the points T and T′ denote the true vacua, the E denotes a solution found in [53]. The arrow connecting T and P denotes a solution of VSSFT representing the original non-BPS brane.

Notion	Parity	GSO	Superghost number	Weight (h)	Comments
A_+	odd	+	1	$h \in \mathbb{Z},\ h \geqslant -1$	string
A_-	even	−	1	$h \in \mathbb{Z}+1/2,\ h \geqslant -1/2$	fields
Λ_+	even	+	0	$h \in \mathbb{Z},\ h \geqslant 0$	gauge
Λ_-	odd	−	0	$h \in \mathbb{Z}+1/2,\ h \geqslant 1/2$	parameters

Table 1: Parity of string fields and gauge parameters in the 0 picture.

2. Cubic vacuum string field theory on a non-BPS D-brane

2.1 Review of cubic string field theory on a non-BPS D-brane

To describe the open string states living on a single non-BPS D-brane one has to consider GSO± states [39]. GSO− states are Grassmann even, while GSO+ states are Grassmann odd (see table 1). The unique (up to rescaling of the fields) gauge invariant cubic action unifying GSO+ and GSO− sectors is [48]

$$S[A_+, A_-] = \frac{1}{g_o^2}\left[\frac{1}{2}\langle\langle Y_{-2}|A_+, Q_B A_+\rangle\rangle + \frac{1}{3}\langle\langle Y_{-2}|A_+, A_+, A_+\rangle\rangle + \right.$$
$$\left. + \frac{1}{2}\langle\langle Y_{-2}|A_-, Q_B A_-\rangle\rangle - \langle\langle Y_{-2}|A_+, A_-, A_-\rangle\rangle\right]. \qquad (2.1)$$

Here the factors before the odd brackets are fixed by the constraint of gauge invariance, that is specified below, and reality of the string fields $A_\pm$.[2] Variation of this action with

[2]Let us mention here that we do not apply restrictions on the string fields which we considered in [48]. One needs to restrict the string fields to ϕ charge 0 and 1 sectors only doing computations within the level truncation scheme.

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respect to A_+, A_- yields the following equations of motion (see [48] for details)

$$\begin{aligned} Q_B A_+ + A_+ \star A_+ - A_- \star A_- &= 0\,, \\ Q_B A_- + A_+ \star A_- - A_- \star A_+ &= 0\,. \end{aligned} \tag{2.2}$$

The action (2.1) is invariant under the gauge transformations

$$\begin{aligned} \delta A_+ &= Q_B \Lambda_+ + [A_+, \Lambda_+] + \{A_-, \Lambda_-\}\,, \\ \delta A_- &= Q_B \Lambda_- + [A_-, \Lambda_+] + \{A_+, \Lambda_-\}\,, \end{aligned} \tag{2.3}$$

where $[\,,]$ ($\{\,,\}$) denotes $\star$-commutator (-anticommutator) and $\Lambda_\pm$ are gauge parameters (see table 1).

The action (2.1) can be rewritten in the matrix form as (see appendix A)

$$S[\hat{A}] = \frac{1}{2g_o^2}\,\mathrm{Tr}\left[\frac{1}{2}\int' \hat{A}\star\hat{Q}_B\hat{A} + \frac{1}{3}\int' \hat{A}\star\hat{A}\star\hat{A}\right], \tag{2.4}$$

where

$$\begin{aligned} \hat{Q}_B &= Q_B \otimes a\,, \qquad \hat{Y}_{-2} = Y_{-2}\otimes a\,, \\ \hat{A} &= A_+ \otimes a + A_- \otimes b \end{aligned} \tag{2.5}$$

and a and b are 2×2 matrices such that

$$a^2 = 1\,, \quad b^2 = -1\,, \quad \{a,b\} = 0\,. \tag{2.6}$$

One can check the following identity

$$\hat{Q}_B(\hat{\Phi}\star\hat{\Psi}) = (\hat{Q}_B\hat{\Phi})\star\hat{\Psi} + (-1)^{|\hat{\Phi}|}\hat{\Phi}\star(\hat{Q}_B\hat{\Psi})\,, \tag{2.7}$$

where $\hat{\Phi}$ and $\hat{\Psi}$ are string fields

$$\hat{\Phi} = \Phi_+\otimes a + \Phi_-\otimes b \qquad \text{and} \qquad \hat{\Psi} = \Psi_+\otimes a + \Psi_-\otimes b \tag{2.8}$$

and parity operator $(-1)^{|\hat{\Phi}|}$ is defined as[3]

$$(-1)^{|\hat{\Phi}|}\Phi_+ = (-1)^{|\Phi_+|}\Phi_+ \qquad \text{and} \qquad (-1)^{|\hat{\Phi}|}\Phi_- = -(-1)^{|\Phi_-|}\Phi_-\,. \tag{2.9}$$

The equation of motion following from the action (2.4) is

$$\hat{Q}_B\hat{A} + \hat{A}\star\hat{A} = 0\,. \tag{2.10}$$

One can check that the equation (2.10) yields the equations (2.2). The action (2.4) is invariant under the following gauge transformations

$$\delta\hat{A} = \hat{Q}_B\hat{\Lambda} + [\hat{A}, \hat{\Lambda}]\,, \tag{2.11}$$

where

$$\hat{\Lambda} = \Lambda_+\otimes 1 + \Lambda_-\otimes ab\,. \tag{2.12}$$

It is a matter of simple algebra to check that the gauge transformations (2.11) yield the gauge transformations (2.3) for the fields $A_\pm$.

[3] Our parity operator (2.9) can also be written as $(-1)^{\mathrm{gh}\,\Phi}$ [58]. This is true because all string fields and gauge parameters which we use satisfy the condition $(-1)^{\mathrm{gh}\,\hat{\Phi}}(-1)^{|\hat{\Phi}|} = 1$.

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Symmetries

The action (2.4) is invariant under the following symmetry transformations:

1. GSO symmetry. It is given by the following transformations

$$\hat{A} \mapsto ((-1)^F \otimes 1)\hat{A}\,, \tag{2.13a}$$

or in components

$$A_+ \mapsto A_+ \qquad \text{and} \qquad A_- \mapsto -A_-\,. \tag{2.13b}$$

One can also check that the BRST charge $\hat{Q}_B$ commutes with $(-1)^F \otimes 1$;

2. Twist symmetry. The generator of this discrete symmetry is denoted by Ω. Its action on the string field is given by the conformal transformation $M(z) = e^{-\pi i} z$. One can easily check that the BRST charge Q_B commutes with Ω.

2.2 Construction of the cubic vacuum superstring field theory

Let $\hat{A}_0$ be a solution of the equation (2.10). A shift of a string field $\hat{A}$

$$\hat{A} = \hat{A}_0 + \hat{\mathcal{A}}\,.$$

yields the following form of the action (2.4)

$$S[\hat{A}_0, \hat{\mathcal{A}}] = S[\hat{A}_0] + \frac{1}{2g_o^2}\,\mathrm{Tr}\left[\frac{1}{2}\int' \hat{\mathcal{A}} \star \hat{Q}\hat{\mathcal{A}} + \frac{1}{3}\int' \hat{\mathcal{A}} \star \hat{\mathcal{A}} \star \hat{\mathcal{A}}\right], \tag{2.14}$$

where $\hat{Q}$ is "a new BRST charge" of the form

$$\hat{Q} = \hat{Q}_B + \{\hat{A}_0, \cdot\}\,. \tag{2.15}$$

Further we will refer to $\hat{Q}$ as to kinetic operator. One can check that the equation $\hat{Q}^2 = 0$ yields the equation of motion for the field $\hat{A}_0$ and therefore $\hat{Q}$ is nilpotent.

Now let us investigate the structure of the shifted BRST charge more carefully. Consider an arbitrary string field $\hat{\Phi}$, then the operator (2.15) acts on it as follows

$$\begin{aligned}\hat{Q}\hat{\Phi} &= Q_B\Phi_+ \otimes 1 + Q_B\Phi_- \otimes ab + \{A_{0,+} \otimes a + A_{0,-} \otimes b, \Phi_+ \otimes a + \Phi_- \otimes b\} \\ &= (Q_B\Phi_+ + \{A_{0,+}, \Phi_+\} - \{A_{0,-}, \Phi_-\}) \otimes 1 + (Q_B\Phi_- + [A_{0,+}, \Phi_-] - [A_{0,-}, \Phi_+]) \otimes ab\,.\end{aligned} \tag{2.16}$$

Let us introduce two new operators Q_{odd} and Q_{even}:[4]

$$Q_{\text{odd}} Z = Q_B Z + A_{0,+} \star Z - (-1)^{|Z|} Z \star A_{0,+}\,, \tag{2.19a}$$

$$Q_{\text{even}} Z = A_{0,-} \star Z + (-1)^{|Z|} Z \star A_{0,-}\,, \tag{2.19b}$$

[4]The generalization of (2.15) and (2.19b) for the case of the arbitrary ghost number of Z was given in [58]:

$$\hat{Q}\hat{Z} = \hat{Q}_B\hat{Z} + \hat{A}_0 \star \hat{Z} - (-1)^{\mathrm{gh}(\hat{Z})}\hat{Z} \star \hat{A}_0 \tag{2.17}$$

and

$$Q_{\text{odd}} Z = Q_B Z + A_{0,+} \star Z - (-1)^{|Z|} Z \star A_{0,+}, \tag{2.18a}$$

$$Q_{\text{even}} Z = A_{0,-} \star Z - (-1)^{\mathrm{GSO}(Z)} Z \star A_{0,-}. \tag{2.18b}$$

This coincides with (2.15) and (2.19b) for odd ghost number.

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where Z is a string field in GSO+ or GSO− sector and $|Z|$ is a parity of the field Z. One sees that (2.16) means that $\hat{Q}$ can be written in the form

$$\hat{Q} = Q_{\text{odd}} \otimes a + Q_{\text{even}} \otimes b\,. \tag{2.20}$$

The nilpotency of the $\hat{Q}$ yields the following identities for the operators Q_{odd} and Q_{even}

$$Q^2_{\text{odd}} - Q^2_{\text{even}} = 0 \qquad \text{and} \qquad [Q_{\text{odd}},\, Q_{\text{even}}] = 0\,. \tag{2.21}$$

Symmetries

Let us now discuss what happened to the symmetries (2.13).

1. GSO symmetry. It is now broken. This happens because the generator of this symmetry $(-1)^F$ does not commute this the kinetic operator (2.15). Under the action of $(-1)^F$ the kinetic operator (2.19), (2.20) transforms to

   $$Q_{\text{odd}} \mapsto Q_{\text{odd}} \qquad \text{and} \quad Q_{\text{even}} \mapsto -Q_{\text{even}}\,.$$

   It is very natural that GSO symmetry is broken when we consider a theory around the vacuum solution. The reason is the following [55]: the original theory has GSO symmetry, therefore if $(A_{0,+}, A_{0,-})$ is a solution of the EOM, then so is $(A_{0,+}, -A_{0,-})$ (see figure 1). And therefore we have to have *two* kinetic operators corresponding to two possible vacuum solutions. This kinetic operators are related to each other by GSO transformation.

2. Twist symmetry. In general the twist symmetry is broken. The only class of the kinetic operators which allows unbroken twist symmetry are the ones corresponding to the solutions satisfying $\Omega(A_{0,+}) = A_{0,+}$ and $\Omega(A_{0,-}) = A_{0,-}$.

2.3 VSSFT Equations of motion

Equations of motion following from the VSFT action (2.14) have the same form as (2.10) but with the shifted BRST operator $\hat{Q}$. In components these equations are

$$\begin{aligned} &Q_{\text{odd}}\mathcal{A}_+ - Q_{\text{even}}\mathcal{A}_- + \mathcal{A}_+ \star \mathcal{A}_+ - \mathcal{A}_- \star \mathcal{A}_- = 0\,,\\ &Q_{\text{odd}}\mathcal{A}_- - Q_{\text{even}}\mathcal{A}_+ + \mathcal{A}_+ \star \mathcal{A}_- - \mathcal{A}_- \star \mathcal{A}_+ = 0\,. \end{aligned} \tag{2.22}$$

It is more convenient to rewrite these equations in "light-cone" variables $\mathcal{A}$ and $\bar{\mathcal{A}}$:

$$\mathcal{A} = \mathcal{A}_+ - \mathcal{A}_- \qquad \text{and} \qquad \bar{\mathcal{A}} = \mathcal{A}_+ + \mathcal{A}_-\,. \tag{2.23}$$

Then equations (2.22) have the following simple form:

$$q\bar{\mathcal{A}} + \mathcal{A} \star \bar{\mathcal{A}} = 0 \qquad \text{and} \qquad \bar{q}\mathcal{A} + \bar{\mathcal{A}} \star \mathcal{A} = 0\,, \tag{2.24}$$

where

$$q = Q_{\text{odd}} - Q_{\text{even}} \qquad \text{and} \qquad \bar{q} = Q_{\text{odd}} + Q_{\text{even}}\,. \tag{2.25}$$

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From the relations (2.21) one can get the following properties of the charges q and $\bar{q}$:

$$q\bar{q} = 0 \qquad \text{and} \qquad \bar{q}q = 0\,. \tag{2.26}$$

It is also fruitful to rewrite the gauge transformation

$$\delta\hat{\mathcal{A}} = \hat{Q}\hat{\Lambda} + [\hat{\mathcal{A}},\, \hat{\Lambda}]$$

in the light-cone variables:

$$\Lambda = \Lambda_+ - \Lambda_- \qquad \text{and} \qquad \bar{\Lambda} = \Lambda_+ + \Lambda_-\,. \tag{2.27}$$

The gauge transformations become

$$\begin{aligned} \delta\mathcal{A} &= q\Lambda + A\star\Lambda - \bar{\Lambda}\star A\,, \\ \delta\bar{\mathcal{A}} &= \bar{q}\bar{\Lambda} + \bar{A}\star\bar{\Lambda} - \Lambda\star\bar{A}\,. \end{aligned} \tag{2.28}$$

3. Construction of the ghost kinetic operators

3.1 Restrictions on $\hat{Q}$ following from Sen conjectures

According to Sen conjectures [39] the solution $\hat{A}_0$ represents the vacuum without open string excitations,[5] and therefore the cohomology of the kinetic operator $\hat{Q}$ must be zero.

As in the bosonic case [1] here it also might be easier to guess the form of the BRST charge then to derive it. In proposing a simple form of the vacuum SSFT action, we have in mind field redefinition, which preserves the form of the cubic action, but simplifies the expression for the kinetic operator $\hat{Q}$. By an appropriate field redefinition

$$\hat{\mathcal{U}} = \mathcal{U}_{\text{even}} \otimes 1 + \mathcal{U}_{\text{odd}} \otimes ab \tag{3.1a}$$

we will assume a $\star$-algebra homomorphism

$$\hat{\mathcal{U}}(\hat{\mathcal{A}}\star\hat{\mathcal{B}}) = (\hat{\mathcal{U}}\hat{\mathcal{A}})\star(\hat{\mathcal{U}}\hat{\mathcal{B}})\,, \tag{3.1b}$$

which satisfy two additional conditions: the invariance of the integral with respect to this homomorphism

$$\mathrm{Tr}\int{}'\,\hat{\mathcal{U}}\hat{\mathcal{A}} = \mathrm{Tr}\int{}'\,\hat{\mathcal{A}}\,; \tag{3.1c}$$

and the existence of the right inverse

$$\hat{\mathcal{U}}\hat{\mathcal{U}}^{-1} = 1\,. \tag{3.1d}$$

The $\hat{}$ in the expressions for the field redefinition $\hat{\mathcal{U}}$ is very important since this transformation acts in both GSO+ and GSO− sectors. Using (3.1) one can check that after the field redefinition

$$\hat{\mathcal{A}} \mapsto \hat{\mathcal{U}}\hat{\mathcal{A}}$$

[5] This conjecture has been checked for the non-BPS brane decay only at the first non-trivial level [38].

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the kinetic operator transforms into

$$\hat{\mathcal{Q}} = \hat{\mathcal{U}}^{-1}\hat{Q}\hat{\mathcal{U}}\,. \tag{3.2}$$

Note that the transformation $\hat{\mathcal{U}}$ is highly non-trivial and mixes GSO+ and GSO− sectors.

Now it can be useful to consider an example of the field redefinition that can seriously simplify an expression for BRST charge. Let consider the standard BRST charge in the superconformal field theory

$$Q_B = \frac{1}{2\pi i}\oint d\zeta\left[c\left(T_B + T_\phi + T_{\eta\xi} + \frac{1}{2}T_{bc}\right) - \eta e^{\phi}T_F + \frac{1}{4}b\partial\eta\eta e^{2\phi}\right]. \tag{3.3}$$

One can check that after the homogenous field redefinition [51]

$$\mathcal{U} = e^{-R}\,, \quad \text{where} \quad R = \frac{1}{2\pi i}\oint d\zeta\left[cT_F e^{-\phi}e^{\chi} + \frac{1}{4}\partial(e^{-2\phi})e^{2\chi}c\partial c\right] \tag{3.4}$$

the BRST charge (3.3) takes the form

$$\mathcal{Q} = \mathcal{U}^{-1}Q_B\mathcal{U} = \frac{1}{2\pi i}\oint d\zeta\, b\gamma^2(\zeta)\,. \tag{3.5}$$

Following the idea of the paper [1], which is based on Sen conjectures, gauge invariance and algebraic properties of the BRST charge, we require $\hat{\mathcal{Q}}$ to satisfy the following properties:

1. $\hat{\mathcal{Q}} = \mathcal{Q}_{\text{odd}}\otimes a + \mathcal{Q}_{\text{even}}\otimes b$;

2. Both $\mathcal{Q}_{\text{odd}}$ and $\mathcal{Q}_{\text{even}}$ have superghost number equal to one, but $\mathcal{Q}_{\text{odd}}$ is Grassmann odd, while $\mathcal{Q}_{\text{even}}$ is Grassmann even;

3. $\hat{\mathcal{Q}}$ is a nilpotent operator, that in components means the identities

$$\mathcal{Q}^2_{\text{odd}} - \mathcal{Q}^2_{\text{even}} = 0 \qquad \text{and} \qquad [\mathcal{Q}_{\text{odd}}, \mathcal{Q}_{\text{even}}] = 0\,; \tag{3.6a}$$

4. $\hat{\mathcal{Q}}$ is a differentiation of the $\star$-algebra

$$\hat{\mathcal{Q}}(\hat{A}\star\hat{B}) = (\hat{\mathcal{Q}}\hat{A})\star\hat{B} + (-1)^{|\hat{A}|}\hat{A}\star(\hat{\mathcal{Q}}\hat{B})\,, \tag{3.6b}$$

   where the parity operator $(-1)^{|\hat{A}|}$ was defined in (2.9). In particular, this identity means that operators $\mathcal{Q}_{\text{odd}}$ and $\mathcal{Q}_{\text{even}}$ also satisfy the Leibnitz rule;

5. The integral of the full derivative is zero

$$\text{Tr}\int' \hat{\mathcal{Q}}(\hat{\mathcal{A}}\star\hat{\mathcal{B}}) = 0\,; \tag{3.6c}$$

6. The operator $\hat{\mathcal{Q}}$ must be universal, what means that it has to be written without reference to the brane boundary CFT;

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7. The operator $\hat{\mathcal{Q}}$ must have vanishing cohomology;

8. The operator $\hat{\mathcal{Q}}$ must commute with the double step inverse picture-changing operator

$$[\hat{Y}_{-2},\, \hat{\mathcal{Q}}] = 0 \quad \text{or} \quad \{\hat{Y}_{-2},\, \hat{\mathcal{Q}}\} = 0\,. \tag{3.6d}$$

   We need this axiom to relate the axiom 5 with the fact that $\hat{\mathcal{Q}}$ annihilates the identity $|\mathcal{I}\rangle$. Therefore we can have several variations of this axiom and in general we only need something like the following

$$\mathcal{Q}_{\rm odd} Y_{-2} \pm Y_{-2}\mathcal{Q}_{\rm odd} = 0 \qquad \text{and} \qquad \mathcal{Q}_{\rm even} Y_{-2} \pm Y_{-2}\mathcal{Q}_{\rm even} = 0\,;$$

   Plus/minus in these formulae can be chosen independently.

9. $\hat{\mathcal{Q}}$ is a hermitian operator, which means that both $Q_{\rm odd}$ and $Q_{\rm even}$ are hermitian ones.

We will construct the operator satisfying these requirements in the next subsection.

Note that in the case of non-polynomial superstring field theory [47] all the requirements remains the same, but the axiom 8 about commutation with the double step inverse picture-changing operator has to be changed into requirement of anticommutation with $\hat{\eta}_0$ (see appendix B for details).

3.2 Construction of the new kinetic operator

The simplest way to satisfy the conditions (3.6) is to put $\mathcal{Q}_{\rm even} = 0$ and choose $\mathcal{Q}_{\rm odd}$ as in the bosonic theory or put

$$\mathcal{Q}_{\rm odd} = \frac{1}{2\pi i}\oint d\zeta\, b\gamma^2(\zeta)\,.$$

However since $A_{0,+} \neq 0$ and $A_{0,-} \neq 0$ we believe that after the field redefinition both charges $\mathcal{Q}_{\rm odd}$ and $\mathcal{Q}_{\rm even}$ are non zero.

One can try to take the following formal expression for the ghost kinetic operator[6]

$$\begin{aligned}
\mathcal{Q}_{\rm odd} &= \mu^2\, c(i) + \frac{1}{2\pi i}\oint b(z)\gamma^2(z)dz\,,\\
\mathcal{Q}_{\rm even} &= \mu\,\gamma(i)\,,
\end{aligned} \tag{3.7}$$

where μ is a complex number. This kinetic operator satisfies the conditions 1-8, however, as it has been noted by Ohmori [58] this operator does not satisfy 9. The following modification of (3.7) has been proposed in [58]

$$\begin{aligned}
\mathcal{Q}_{\rm odd} &= \frac{\mu^2}{4i}\left[c(i) - c(-i)\right] + \frac{1}{2\pi i}\oint b(z)\gamma^2(z)dz\,,\\
\mathcal{Q}^+_{\rm even} &= \frac{\mu}{2i}\left[\gamma(i) - \gamma(-i)\right],\\
\mathcal{Q}^-_{\rm even} &= \frac{\mu}{2}\left[\gamma(i) + \gamma(-i)\right],
\end{aligned} \tag{3.8}$$

[6] We have discussed this form of the ghost kinetic operator in the first version of this paper [57].

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where $\mathcal{Q}^{\pm}_{\rm even}$ means the restriction of the operator $\mathcal{Q}_{\rm even}$ to GSO$\pm$ sectors. In some sense (3.8) is the only form for the kinetic operator which satisfies the twist invariance and the conditions (3.6). One can explain it as follows. Following [24] consider an original (before field redefinition) BRST charge Q defined as

$$Q = \sum_r \frac{1}{2\pi i} \oint d\zeta \, a_r(\zeta) \mathcal{O}_r(\zeta) \,, \tag{3.9}$$

where a_r are smooth forms of ζ and $\mathcal{O}_r(\zeta)$ are some local conformal operators of ghost number 1. It was shown [24] that after a singular field redefinition the dominant contribution to the transformed charge $\mathcal{Q}$ will come from the lowest dimensional conformal operators. This has led to the choice of $c(i)$ and $c(-i)$ in the bosonic case, and this also leads to our choice of $\mathcal{Q}_{\rm even}$, since γ is the lowest dimensional even primary operator of ghost number 1.

The ansatz (3.8) obviously satisfies the axiom 2 from the previous subsection. The axiom 3 about nilpotency of $\hat{\mathcal{Q}}$ is also satisfied, because

$$\mathcal{Q}^2_{\rm odd} \equiv \mathcal{Q}^{\mp}_{\rm even} \mathcal{Q}^{\pm}_{\rm even} = \frac{\mu^2}{4i} (\gamma^2(i) - \gamma^2(-i)) \tag{3.10a}$$

and since there is no β in the expression for $\mathcal{Q}_{\rm odd}$ one gets

$$[\mathcal{Q}_{\rm odd} \,,\, \mathcal{Q}_{\rm even}] = 0 \,. \tag{3.10b}$$

The most non trivial is to check the axiom 4, which in particular says that $\mathcal{Q}_{\rm odd}$ and $\mathcal{Q}_{\rm even}$ admit the Leibnitz rule and the axiom 5, which says that integral of "the full derivative" is zero. These axioms are also related to the hermitian property of the kinetic ghost operator

$$\mathrm{Tr}\, \langle\langle \hat{\mathcal{A}},\ \hat{\mathcal{Q}}\hat{\mathcal{B}} \rangle\rangle = \mathrm{Tr}\, \langle\langle \hat{\mathcal{Q}}\hat{\mathcal{A}},\ \hat{\mathcal{B}} \rangle\rangle \tag{3.11}$$

and the gauge invariance of the vacuum string field theory action. It is a matter of overlap equations that if we could proof the axiom 5, then the axiom 4 will be automatically satisfied.

3.2.1 Check of axiom 5

It doesn't seem that $\mathcal{Q}_{\rm odd}$ and $\mathcal{Q}_{\rm even}$, which contain midpoint insertions, can satisfy axiom 5. The reason is that they diverge acting on the identity $|\mathcal{I}\rangle$. However, one can define $\mathcal{Q}_{\rm odd}$ and $\mathcal{Q}_{\rm even}$ as a limit of a sequence with each element annihilating the identity.

To construct such a sequence let us consider the overlap equation for the identity. To write it one has to be very careful, because in the superconformal CFT we have conformal fields with half integer weights. We will also assume that we make double trick for antiholomorphic operators, and therefore the overlap equation will connect the fields on the boundary of the unit disk, and not only on the boundary of the upper half unit disk. The argument of coordinate z on the unit disk is in interval $(-\pi, \pi)$. The following overlap

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equations for a conformal operator $\mathcal{O}_h$ of the weight h take place:

$$\left[\mathcal{O}_h(z) - \left(\frac{1}{z^2}\right)^h \mathcal{O}_h\left(\frac{e^{i\pi}}{z}\right)\right]|\mathcal{I}\rangle = 0\,,\ \text{for}\,|z|=1\,,\ \mathrm{Re}\,z<0\,\text{and}\,\mathrm{Im}\,z>0\,;$$
$$\left[\mathcal{O}_h(z) - \left(\frac{e^{-2\pi i}}{z^2}\right)^h \mathcal{O}_h\left(\frac{e^{-i\pi}}{z}\right)\right]|\mathcal{I}\rangle = 0\,,\ \text{for}\,|z|=1\,,\ \mathrm{Re}\,z<0\,\text{and}\,\mathrm{Im}\,z<0\,. \quad (3.12)$$

So one sees that if h is a half integer the second equation differs from the first by a sign of the second term.

Now the regularization is clear. Let us simply define kinetic operators $\mathcal{Q}^{\varepsilon}_{\text{odd}}$ and $\mathcal{Q}^{\varepsilon}_{\text{even}}$ by substitutions

$$c(i) \mapsto \frac{1}{2}\left[e^{-i\varepsilon}c(ie^{i\varepsilon}) + e^{i\varepsilon}c(ie^{-i\varepsilon})\right],$$
$$\gamma(i) \mapsto \frac{1}{e^{-i\pi/4} - e^{i\pi/4}}\left[e^{-i\pi/4-i\varepsilon/2}\gamma(ie^{i\varepsilon}) - e^{i\pi/4+i\varepsilon/2}\gamma(ie^{-i\varepsilon})\right]. \quad (3.13)$$

This regularization corresponds to a splitting of the midpoint insertion into two insertions on the left-half and on the right-half of the string. In limit $\varepsilon \to 0$ one recovers the midpoint insertion. From overlap equations (3.13) it follows that

$$\left[e^{-i\varepsilon}c(ie^{i\varepsilon}) + e^{i\varepsilon}c(ie^{-i\varepsilon})\right]|\mathcal{I}\rangle = 0\,,$$
$$\left[e^{-i\pi/4-i\varepsilon/2}\gamma(ie^{i\varepsilon}) - e^{i\pi/4+i\varepsilon/2}\gamma(ie^{-i\varepsilon})\right]|\mathcal{I}\rangle = 0\,. \quad (3.14)$$

One can now define $\mathcal{Q}_{\text{odd}}$ and $\mathcal{Q}_{\text{even}}$ as

$$\mathcal{Q}_{\text{odd}} \equiv \lim_{\varepsilon\to 0}\mathcal{Q}^{\varepsilon}_{\text{odd}} \qquad \text{and} \qquad \mathcal{Q}_{\text{even}} \equiv \lim_{\varepsilon\to 0}\mathcal{Q}^{\varepsilon}_{\text{even}}\,. \quad (3.15)$$

This finishes the proof that $\mathcal{Q}_{\text{odd}}$ and $\mathcal{Q}_{\text{even}}$ annihilate the identity.

3.2.2 Check of axiom 7. Zero cohomology

As in the bosonic case [24] the equation $\hat{\mathcal{Q}}\hat{\Phi} = 0$ has no non-zero solutions which belong to Fock space.[7] Let us suppose that there is a generalized state $\hat{\Psi}$ annihilated by $\hat{\mathcal{Q}}$. We want to show that for any such $\hat{\Psi}$ there exists $\hat{\Lambda}$ such that

$$\hat{\Psi} = \hat{\mathcal{Q}}\hat{\Lambda}\,. \quad (3.16)$$

To show this it is sufficient to find an operator $\hat{\mathcal{K}}$ such that

$$\{\hat{\mathcal{Q}},\,\hat{\mathcal{K}}\} = \mathrm{Id}\otimes 1\,. \quad (3.17)$$

Indeed, acting by $\hat{\mathcal{K}}\hat{\mathcal{Q}}$ onto the expression (3.16)

$$0 = \hat{\mathcal{K}}(\hat{\mathcal{Q}}\hat{\Psi}) \overset{(3.17)}{=} \hat{\Psi} - \hat{\mathcal{Q}}(\hat{\mathcal{K}}\hat{\Psi})\,,$$

we find $\hat{\Lambda} = \hat{\mathcal{K}}\hat{\Psi}$.

[7] This is so because $\hat{\mathcal{Q}}$ involves oscillators of all possible levels, while states in Fock space have to be polynomials.

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It is natural to search for the operator $\hat{\mathcal{K}}$ in the following form

$$\hat{\mathcal{K}} = \mathcal{K}_+ \otimes a + \mathcal{K}_- \otimes b\,. \tag{3.18}$$

The equation (3.17) yields

$$\mathrm{Id} \otimes 1 = \{\hat{\mathcal{Q}}, \hat{\mathcal{K}}\} = \big(\{\mathcal{Q}_{\rm odd}, \mathcal{K}_+\} - \{\mathcal{Q}_{\rm even}, \mathcal{K}_-\}\big) \otimes 1 + \big(-[\mathcal{Q}_{\rm even}, \mathcal{K}_+] + [\mathcal{Q}_{\rm odd}, \mathcal{K}_-]\big) \otimes ab\,.$$

For the operators $\mathcal{Q}_{\rm odd}$ and $\mathcal{Q}_{\rm even}$ defined by (3.8) one can take

$$\mathcal{K}_- = 0 \qquad \text{and} \qquad \mathcal{K}_+ = \frac{2}{\mu^2}\, b_0\,. \tag{3.19}$$

3.2.3 Check of axiom 8

We show that the following relations are true

$$[\mathcal{Q}_{\rm odd}, Y_{-2}] = 0 \qquad \text{and} \qquad [\mathcal{Q}_{\rm even}, Y_{-2}] = 0\,. \tag{3.20}$$

The first commutator is obviously zero, since Y_{-2} commutes with $c(\cdot)$. It also commutes with $\oint b\gamma^2$, since it is a part of the original BRST charge. Let us check the second commutator. To this end let us remind, that

$$Y_{-2} = Y(i)Y(-i)\,, \qquad Y(z) = 4c\partial\xi e^{-2\phi}(z) \qquad \text{and} \qquad \gamma(z) = \eta e^{\phi}(z)\,.$$

Consider the following OPEs:

$$\begin{aligned} Y(i)Y(-i)\gamma(z) \;=\; & -c(i)c(-i)\Big[:\partial\xi(i)\partial\xi(-i)\eta(z): - \frac{\partial\xi(i)}{(i+z)^2} + \frac{\partial\xi(-i)}{(i-z)^2}\Big] \times \\ & \times (z^2+1)^2 e^{-2\phi(i)-2\phi(-i)+\phi(z)} \\ \gamma(z)Y(i)Y(-i) \;=\; & -c(i)c(-i)\Big[:\partial\xi(i)\partial\xi(-i)\eta(z): - \frac{\partial\xi(i)}{(i+z)^2} + \frac{\partial\xi(-i)}{(i-z)^2}\Big] \times \\ & \times (z^2+1)^2 e^{-2\phi(i)-2\phi(-i)+\phi(z)}\,. \end{aligned}$$

So one sees that $[\gamma(z), Y_{-2}] = 0$ for any z. Now assuming that we are dealing with the regularized charge $\mathcal{Q}^{\varepsilon}_{\rm even}$ we get that it commutes with the double step inverse insertion operator. And after taking the limit $\varepsilon \to 0$ we obtain the result required for $\mathcal{Q}_{\rm even}$.

3.2.4 Check of axiom 9

It has been shown in [58] that the hermitian property (3.11) holds due to the following conformal properties of the kinetic operator

$$\begin{aligned} \mathcal{Q}^{+}_{\rm even} &= -I \circ \mathcal{Q}^{-}_{\rm even}\,, \\ \mathcal{Q}^{-}_{\rm even} &= I \circ \mathcal{Q}^{+}_{\rm even}\,, \end{aligned} \tag{3.21}$$

where $I(z) = -1/z$.

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3.2.5 Symmetries

Let us now check that the ghost kinetic operators (3.8) satisfy the symmetries we have discussed on page 6.

1. GSO symmetry. GSO transformation relates the ghost kinetic operators (3.8) with μ and $-\mu$.

2. Twist symmetry. Twist transformation Ω changes $i \mapsto -i$ and

$$\begin{aligned} \Omega c(i)\Omega^{-1} &= -c(-i)\,, \qquad \Omega c(-i)\Omega^{-1} = -c(i)\,, \\ \Omega\gamma(i)\Omega^{-1} &= i\,\gamma(-i)\,, \qquad \Omega\gamma(-i)\Omega^{-1} = i\,\gamma(i)\,. \end{aligned} \tag{3.22}$$

   It has been shown by Ohmori [58] that due to [48]

$$\begin{aligned} \Omega(|\mathcal{A}_+\rangle) &= (-1)^{h_{\mathcal{A}_+}+1}|\mathcal{A}_+\rangle\,, \\ \Omega(|\mathcal{A}_-\rangle) &= (-1)^{h_{\mathcal{A}_-}+\frac{1}{2}}|\mathcal{A}_-\rangle\,, \end{aligned} \tag{3.23}$$

   the following properties on homogenous strings fields hold

$$\begin{aligned} \Omega(\mathcal{Q}^+_{\text{even}}|\mathcal{A}_+\rangle) &= \mathcal{Q}^+_{\text{even}}\Omega(|\mathcal{A}_+\rangle)\,, \\ \Omega(\mathcal{Q}^-_{\text{even}}|\mathcal{A}_-\rangle) &= \mathcal{Q}^-_{\text{even}}\Omega(|\mathcal{A}_-\rangle)\,. \end{aligned} \tag{3.24}$$

   This means that the twist invariant choice of the kinetic operator $\mathcal{Q}_{\text{even}}$ takes the different form in GSO+ and GSO− sectors.

4. Conclusion and discussions

In this paper we have considered a simplest candidate for the kinetic operator of the cubic VSSFT which describes a result of a decay of unstable non-BPS brane. In spite of the fact that we consider only cubic superstring field theory, our results concerning the ghost kinetic operator can be applied to the Berkovits non-polynomial superstring field theory (see appendix B).

Let us also note that there are subtleties relating to the insertion of both the kinetic operators and the picture changing operators in the string midpoint, so that a part of the action naively vanishes. The resolution of these subtleties may be in finding a suitable regularization for ghost kinetic operators (see section 3.2.1). A singular nature of the VSFT action and the sliver solution is known, but it have not been known yet how to regularize them properly. Since our nearest aim is to solve VSFT equations of motion [45, 56] which do not contain the picture changing operator we can ignore this problem for the moment. One has to find a proper regularization trying to study the brane tension problem in VSSFT.

As the problems to be solved let us note

- Find the solution of the ghost equations of motion;[8]
- Question about the restoration of the supersymmetry in the non-perturbative vacuum.

[8] Solutions of the ghost equations of motion has been discussed recently in [56], [58]

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Acknowledgments

We would like to thank A.S. Koshelev for discussions. This work was supported in part by RFBR grant 02-01-00695 and RFBR grant for leading scientific schools and by INTAS grant 99-0590.

A. Matrix representation of non-GSO projected string fields

The cubic action involving these two sectors was constructed by the authors in [48]. Here we will rewrite it in more simple form using only one (matrix valued) string field:

$$\hat{\mathcal{A}} = \mathcal{A}_+ \otimes a + \mathcal{A}_- \otimes b\,, \tag{A.1}$$

where a and b are $N \times N$ matrices we wish to find. We have to write also

$$\hat{Q}_B = Q_B \otimes q \qquad \text{and} \qquad \hat{Y}_{-2} = Y_{-2} \otimes y\,, \tag{A.2}$$

where q and y are also $N \times N$ matrices we need to determine. We propose the action for string field (A.1) in the form

$$S = \frac{1}{g_o^2}\frac{1}{N}\,\mathrm{Tr}\left[\frac{1}{2}\int' \hat{\mathcal{A}} \star \hat{Q}_B \hat{\mathcal{A}} + \frac{1}{3}\int' \hat{\mathcal{A}} \star \hat{\mathcal{A}} \star \hat{\mathcal{A}}\right], \tag{A.3}$$

where Tr is trace over the matrix multiplier and $\int'$ denotes Witten's integration but with the insertion of double step inverse picture changing operator Y_{-2}, i.e. $\int' = \int \hat{Y}_{-2}$. One can check that to obtain the action proposed in [48] the matrices q, y, a, b have to satisfy the following conditions:

$$\mathrm{Tr}(yaqa) = \mathrm{Tr}(ybqb) = N\,; \tag{A.4a}$$

$$\mathrm{Tr}(ya^3) = N\,; \tag{A.4b}$$

$$\mathrm{Tr}(yab^2) = -\,\mathrm{Tr}(ybab) = \mathrm{Tr}(yb^2a)\,; \tag{A.4c}$$

$$\mathrm{Tr}(yab^2) = -N\,. \tag{A.4d}$$

The condition (A.4c) is satisfied if

$$[a, b^2] = 0 \qquad \text{and} \qquad \{a, b\} = 0\,. \tag{A.5a}$$

The condition (A.4b) is satisfied if

$$a^2 = 1 \qquad \text{and} \qquad y = a\,. \tag{A.5b}$$

And the conditions (A.4a) and (A.4d) are satisfied if

$$q = a \qquad \text{and} \qquad b^2 = -1\,. \tag{A.5c}$$

So we are left with two matrices a and b such that

$$a^2 = 1\,, \qquad b^2 = -1 \qquad \text{and} \qquad \{a, b\} = 0\,. \tag{A.6}$$

Further we will assume that a and b are 2×2 matrices, for example

$$a = \sigma_3\,, \quad b = i\sigma_2 \qquad \text{and} \qquad ab = \sigma_1\,. \tag{A.7}$$

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B. Application to non-polynomial string field theory

Let us remind the action for the NS sector of the non-polynomial super string field theory on a non-BPS brane [47]:

$$S[\hat{\Phi}] = \frac{1}{4}\,\mathrm{Tr}\int\left[(e^{-\hat{\Phi}}\hat{Q}_B e^{\hat{\Phi}})(e^{-\hat{\Phi}}\hat{\eta}_0 e^{\hat{\Phi}}) - \int_0^1 dt\,(e^{-t\hat{\Phi}}\partial_t e^{t\hat{\Phi}})\{(e^{-t\hat{\Phi}}\hat{Q}_B e^{t\hat{\Phi}}),\,(e^{-t\hat{\Phi}}\hat{\eta}_0 e^{t\hat{\Phi}})\}\right], \tag{B.1}$$

where

$$\hat{\Phi} = \Phi_+ \otimes 1 + \Phi_- \otimes ab\,, \quad \hat{Q}_B = Q_B \otimes a \quad \text{and} \quad \hat{\eta}_0 = \eta_0 \otimes a\,. \tag{B.2}$$

Here matrices a and b are defined by (A.7), Φ_+ and Φ_- are picture zero superghost zero string fields in GSO+ (Grassmann even) and GSO− (Grassmann odd) sector respectively

It is shown in the papers [54] and [46] that the action for the shifted string field $\hat{h}$ has the same form as the original one (B.1), but with the shifted BRST charge $\hat{Q}$, which is defined on an arbitrary string field $\hat{\Phi}$ as

$$\hat{Q}\hat{\Phi} = \hat{Q}_B\hat{\Phi} + [\hat{A},\,\hat{\Phi}]\,. \tag{B.3}$$

Here

$$\hat{A} = e^{-\hat{\Phi}_0}\hat{Q}_B e^{\hat{\Phi}_0} \tag{B.4}$$

and Φ_0 is a solution of the equations of motion following from (B.1). Using the matrix representations for $\hat{Q}_B$ and $\hat{\Phi}_0$ one can show that $\hat{A}$ can be represented in the following form

$$\hat{A} = A_{\mathrm{odd}} \otimes a + A_{\mathrm{even}} \otimes b\,. \tag{B.5}$$

Now let us substitute this representation into (B.3)

$$\begin{aligned}\hat{Q}\hat{\Phi} &= Q_B\Phi_+ \otimes a + Q_B\Phi_- \otimes b + [A_{\mathrm{odd}} \otimes a + A_{\mathrm{even}} \otimes b,\,\Phi_+ \otimes 1 + \Phi_- \otimes ab] \\ &= (Q_B\Phi_+ + [A_{\mathrm{odd}},\,\Phi_+] + \{A_{\mathrm{even}},\,\Phi_-\}) \otimes a + \\ &\quad + (Q_B\Phi_- + \{A_{\mathrm{odd}},\,\Phi_-\} + [A_{\mathrm{even}},\,\Phi_+]) \otimes b\,.\end{aligned} \tag{B.6}$$

Let us introduce two new operators Q_{odd} and Q_{even}:

$$Q_{\mathrm{odd}}X = Q_B X + A_{\mathrm{odd}} \star X - (-1)^{|X|}X \star A_{\mathrm{odd}}\,, \tag{B.7a}$$

$$Q_{\mathrm{even}}X = A_{\mathrm{even}} \star X - (-1)^{|X|}X \star A_{\mathrm{even}}\,, \tag{B.7b}$$

where X is a string field in GSO+ or GSO− sector and $|X|$ is a parity of the field X. Notice that the difference between the formulae (2.19b) and (B.7b) in the sign is only due to the difference in the definition of the string field (compare (2.5) and (B.2)), but in principle the operators defined by (B.7) and (2.19) are the same. One sees that (B.6) means that $\hat{Q}$ can be written in the form

$$\hat{Q} = Q_{\mathrm{odd}} \otimes a + Q_{\mathrm{even}} \otimes b\,. \tag{B.8}$$

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The nilpotency[9] of the $\hat{Q}$ yields the following identities for the operators $Q_{\rm odd}$ and $Q_{\rm even}$

$$Q_{\rm odd}^2 - Q_{\rm even}^2 = 0 \qquad \text{and} \qquad [Q_{\rm odd},\, Q_{\rm even}] = 0\,. \tag{B.9}$$

Starting from this moment all the results obtained in section 3 can be applied to the BRST charge defined by (B.8) without any modifications. Moreover one can check that the ghost kinetic operator (3.8) satisfies the equation $\{\hat{\eta}_0,\, \hat{\mathcal{Q}}\} = 0$.

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[9] Let us notice the difference in cubic and non-polynomial SFTs. In the cubic SFT the nilpotency of the shifted BRST charge yields the equations of motion for the background field, while in the non-polynomial one the nilpotency of the shifted BRST charge is just an algebraic identity. The equations of motion for the background field are obtained from the requirement $\{\eta_0,\, Q\} = 0$. Therefore one has to add this condition to the axioms in the section 3.

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